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# Maze Solving using Fatty Acid Chemistry

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TITLE RUNNING HEAD

Maze solving

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**Abstract**

This study demonstrates that the Marangoni flow in a channel network can solve maze problems such as exploring and visualizing the shortest path and finding all possible solutions in a parallel fashion. The Marangoni flow is generated by the pH gradient in a maze filled with an alkaline solution of a fatty acid by introducing a hydrogel block soaked with an acid at the exit. The pH gradient changes the protonation rate of fatty acid molecules that translates into the surface tension gradient at the liquid-air interface through the maze. Fluid flow maintained by the surface tension gradient (Marangoni flow) can drag water soluble dye particles towards low pH (exit) at the liquid-air interface. Dye particles put to the entrance of the maze dissolve during the motion thus visualizing and finding the shortest path and all possible paths in a maze.

## KEYWORDS

Maze solving, Marangoni flow, surface tension, fatty acid, shortest path problem

## INTRODUCTION

Efficient maze solving, i.e. finding either the shortest path or all possible paths between two points in a maze, is a challenging mathematical problem especially if computational constraints are imposed. These problems can be solved using inherently slow computational methods where the solving time,  $t$ , scales with the size of the system,  $n$ , raised to some power  $x$ , i.e.  $t : n^x$ .<sup>1,2</sup> However, there are other possibilities such as analog computational methods that have been successfully used to solve these problems. They rely either solely on maze topology (examples include applying a pressure difference in fluid through the maze,<sup>3</sup> processing the path of propagating chemical waves<sup>4</sup> or plasma,<sup>5</sup> using a network of memristors<sup>6</sup> and the reconfiguration of an organism between two food sources within the maze<sup>7,8,9</sup>) or on a path-finding entity (solid hydrogel in a microfluidic network or droplet suspended at the liquid-air interface) that can react to environmental stimuli.<sup>10,11</sup> In the latter example, it has been shown that a small fatty acid droplet suspended at the liquid-air interface in an alkaline solution can

solve the maze and find the shortest path if a pH gradient is established in the maze.

Using these aforementioned analog computing methods can be inherently slow due to the nature of the processes used (e.g., chemical wave propagation,<sup>4</sup> protoplasmic tube formation<sup>7,8,12</sup>). Additionally, in most cases only one path (the shortest) can be explored and/or visualized. It should be mentioned that using a memristive network all paths can be find and the maze solving time could be short (a few nanoseconds).<sup>6</sup> However, chemical-based computing methods providing all possible paths trough a maze are lacking and challenging.

In this paper we will provide a new method to find and visualize the shortest and all possible paths in a maze based on a Marangoni flow<sup>13-20</sup> induced by a surface tension gradient at the liquid-air interface<sup>21</sup>. Specifically, we describe one such system that comprises small tracer (solid water soluble dye) particles powered by an acid/base reaction. When subject to a gradient of pH within a maze, these particles move towards regions of low pH and find the shortest path. Due to water solubility of dye particles the path can be visualized and all possible paths can be explored.

## EXPERIMENTAL SECTION

We investigated the maze solving in mazes with different complexity made from polydimethylsiloxane (PDMS) that were designed by photo- and laserlithography. The size of the channel was 1.4 mm thick and 1.0 mm deep. In experiments, we first filled the maze with an aqueous solution of KOH (Sigma-Aldrich, 0.05 M) containing 0.2 v% of 2-Hexyldecanoic acid (Sigma Aldrich) (2-HDA). A small block of agarose gel (Sigma-Aldrich) soaked in a solution of HCl (Sigma-Aldrich, 1.0 M) was placed at one entrance (exit) of the maze. Immediately after the addition of acidic block, a small amount (~ 0.3 mg) of Phenol Red dye powder was placed at the liquid-air interface at the other entrance to the maze. The dye particles located at the interface moved towards the regions of lower pH – that is, towards the source of HCl. During the transport the dye particles dissolved in liquid phase in a maze thus visualizing the path of those particles.



## MODELING

In order to model this phenomenon, namely the fluid flow in a 2D cross-section in a channel, we considered the fully developed, steady flow within the 2D rectangular domain  $\Omega \subset \mathbb{R}^2$  driven by a spatially non-uniform surface tension,  $\gamma$ , at the liquid-air interface. The fluid flow within the domain is described by the non-dimensionalized Navier-Stokes equation<sup>22</sup>

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + Re^{-1} \Delta \mathbf{u}, \quad \nabla \cdot \mathbf{u} = 0, \quad (1)$$

where  $\mathbf{u} = (u_x(x, y), u_y(x, y))$  is the two-dimensional velocity field, and  $Re = UL/\nu$  is the Reynolds number with the characteristic horizontal length  $L$  and velocity  $U$ , and the dynamic viscosity  $\nu$ . The Reynolds number can be interpreted as a measure of importance of advection against diffusion of the (linear) momentum of the fluid in the dynamics. The Marangoni-type flow considered here is driven by the non-uniform surface tension on the top (liquid-air) boundary (assumed rigid) which leads to the following constraints on the tangential stresses:<sup>23</sup>

$$\frac{\partial u_x}{\partial y} = \frac{\partial \hat{\gamma}}{\partial x}, \quad u_y = 0, \quad (2)$$

where  $\hat{\gamma}$  is the dimensionless surface tension and the boundary is decomposed as  $\partial\Omega \equiv \partial\Omega_T \cup \partial\Omega_0$ , where  $\partial\Omega_T$  denotes the top surface and  $\partial\Omega_0 = \partial\Omega \setminus \partial\Omega_T$ ; the remaining boundary conditions are 'no-slip', i.e.,  $\mathbf{u}(x, y) = 0$  on  $\partial\Omega_0$ . By taking advantage of the 2D approximation, the equation (1) can be represented as

$$\frac{\partial \nabla^2 \psi}{\partial t} + \frac{\partial (\nabla^2 \psi, \psi)}{\partial (x, y)} = Re^{-1} \nabla^4 \psi, \quad (3)$$

where the scalar streamfunction,  $\psi$ , is related to the velocity via  $\mathbf{u} = (\partial_y \psi, -\partial_x \psi)$ ,

$\partial(\nabla^2\psi, \psi)/\partial(x, y) \equiv \partial_x \nabla^2\psi \partial_y \psi - \partial_y \nabla^2\psi \partial_x \psi$  with  $\partial_x \equiv \partial/\partial x$ ,  $\partial_y \equiv \partial/\partial y$ . In the streamfunction representation the boundary conditions become

$$\psi = 0 \text{ on } \partial\Omega, \quad \mathbf{n}_{\partial\Omega} \cdot \nabla \psi = 0 \text{ on } \partial\Omega, \text{ and } \partial_y^2 \psi = \partial_x \hat{\gamma} \text{ on } \partial\Omega_T. \quad (4)$$

We look for a fully developed steady solution of (3) which satisfies

$$\frac{\partial(\nabla^2\psi, \psi)}{\partial(x, y)} = Re^{-1} \nabla^4 \psi, \quad (5)$$

with the same boundary conditions as in Eq. (4). The numerical solutions to the nonlinear problem (Eqs. 4-5) are found by using a simple second-order, finite-difference discretization on a structured, equidistant grid which is combined with the Newton-Raphson iteration in order to find the solution; the biharmonic term,  $\nabla^4 \psi$ , in Eq. (5) is discretized using the 13-point stencil and the Laplacian in Eq. (5) is discretized using the standard 9-point stencil. We suppose that the surface tension at the liquid-air interface can be approximated by the following functional form (Figure 2a)

$$\hat{\gamma} = \tanh(a(x - x_0)) / \left( \tanh(a(1/2 - x_0)) + \tanh(a(1/2 + x_0)) \right), \quad (6)$$

where  $a$  and  $x_0$  are constants. We can use this approximation, because the surface tension is determined by the concentration of deprotonated 2-HDA. This depends on pH, and in our experimental setup, the distribution of the pH ( $H^+$ ) has sigmoidal shape, because an acidic source continuously releases  $H^+$  ions into an alkaline solution (where the concentration of  $H^+$  is low). In alkaline solution the amount of deprotonated form dominates providing lower surface tension compared to the surface tension in acidic solution, in which the protonated form exists predominantly and the protonated form is insoluble in water. So, in other words, the distribution of pH translates into the distribution of the surface tension. We should point out that to obtain a more precise distribution of the surface tension would involve solving a set of coupled reaction-diffusion-convection equations for all species. This complicates the model and its numerical solution. Our aim was to reproduce the main behavior observed in experiments, so we use Eq. (5) as a first guess approximation to estimate the surface tension profile at the liquid-air

interface.

## RESULTS AND DISCUSSION

Figure 1a shows the Marangoni flow induced maze solving using various mazes with different location of the entrances and exits. In these mazes with relatively simple topology the shortest path can be visualized within  $\sim 10$  s using water soluble dye particles. Maze solving by particles can be explained by surface tension originating from non-uniform distribution of the deprotonated form of 2-HDA at the liquid-air interface. Initially the maze is filled with an alkaline solution, 2-HDA molecules are homogeneously distributed and deprotonated at the liquid-air interface. By introducing acidic block at one side,  $H^+$  ions diffuse from the block establishing a pH gradient that protonates non-uniformly the fatty acid molecules according to a reversible pH dependent reaction  $H^+ + DA^- \rightleftharpoons HDA$ , where  $DA^-$  and  $HDA$  are the deprotonated and protonated forms of 2-HDA, respectively. Due to this reversible reaction the pH directly determines the concentration of the protonated and deprotonated forms of 2-HDA.<sup>24</sup> Importantly, less deprotonated 2-HDA is found in the direction facing the source of acid (i.e., towards lower pH). This asymmetric distribution translates into a gradient of surface tension which is determined predominantly by the concentration of the deprotonated 2-HDA at the liquid-air interface. The surface tension gradient gives rise to the convective flow (towards low pH at the interface and opposite flow at the bottom of the channel, Figure 2a and Movie 1 in Supplementary Information), which is ultimately responsible for particle motion, thus visualizing the shortest path.

Figure 2 presents the generated convection roll in a channel (cross section) obtained from experiment and numerical simulation maintained by the Marangoni flow due to the inhomogeneous distribution of the deprotonated HDA at the interface. In experiments we observed that the average velocity of a small tracer particle (here we used a small piece of a PDMS of size  $1.0 \times 0.5 \times 0.5$  mm) are  $\sim 1.3$  mm/s (Figure 2a and Movie 1 in Supplementary Information). However, it should be mentioned that the velocity of a tracer at the liquid-air interface is lower when further away from the acidic block. When it

flows closer and approaches the acidic block because of convection, the velocity increases up to a maximum value ( $\sim 3$  mm/s). The simplified numerical model also captures this behavior.

Simulating the transport of passive tracer particles through the maze, would involve solving the Navier-Stokes equation in 3D (Eq. 1 - describing the velocity field) and convection-reaction-diffusion equations (describing the spatial variation of the protonated and deprotonated form of 2-HDA and pH ( $H^+$ )) equipped with an equation  $\gamma = \gamma(DA^-)$  that determines the surface tension dependence on the deprotonated form of 2-HDA. Instead of carrying out these complicated (due to the geometry of the domain and boundary conditions) and time consuming (due to 3D representation) numerical simulations, we can assume that the shortest path and all other possible paths through a maze can be obtained by drawing the streamlines of the gradient vector field of the concentration (which correlate with the highest surface tension gradient at a given point) at the liquid-air interface in a network of channels in 2D via solving the Laplace equation  $\nabla^2 c = 0$ , where  $c$  is the concentration of  $DA^-$ . Particles move towards the highest surface tension gradient that is equivalent to the highest concentration gradient, thus each path can be represented as a streamline of the concentration gradient field. The concentration of the deprotonated form in a maze monotonically decreases with respect to a path length from a starting point (entrance of the maze). The stationary concentration distribution in a maze was calculated using the finite difference method on a rectangular mesh. The boundary conditions were the Neumann no-flux conditions for the wall of the maze and the Dirichlet conditions for the entrance and exit of the maze. For simplicity we used  $c = 1$  at the entrance and  $c = 0$  at the exit, which represent high and very low (virtually zero) concentration of  $DA^-$  at the entrance and at the exit, respectively. Figure 1b shows the obtained streamlines from the numerical simulations, it can be seen that most of these streamlines follow the shortest paths in various mazes thus solving the maze problem.

Importantly, in our experiments the particles always find the shortest path through the maze, however, with increasing time, also the longer alternative paths are found. Figure 3 presents this situation in a

topologically more complex maze than presented in Figure 1. Figure 3a shows the visualized shortest path in a maze after  $\sim 10$  s. There are always several streams in channels in the maze connected and interconnected with the acidic block (exit) that maintains the fluid flow. The intensity of these streams is inversely proportional to the length of the paths. In other words, the most intense fluid flow is established along a path in a maze, where the gradient of the surface tension at the liquid-air interface is the highest. Therefore, the most intense stream is realized in the shortest path that carries most of the dye particles thus the shortest path is visualized at first. Less intense flow can carry less particles, therefore, visualization of longer paths needs proportionally more time. However, eventually all possible paths would be explored and visualized due to dissolution of solid dye particles (Figure 3b,c). It should be also mentioned that some dye particles can migrate into dead-end channels (Figure 1a, 3a). This could be attributed to the fact that the generated fluid flow in a maze can induce some fluid motion in those channels in which no surface tension gradient is established due to the inertia of the fluid flow.

To illustrate our maze finding concept in a realistic configuration, we fabricated a maze by analogy to the street map of downtown of Budapest (Figure 4). In this setup, similarly to former experiments, we filled the maze with an alkaline solution of 2-HDA and added a solid hydrogel block soaked with acid to the end point (place to find – indicated by B in the Figure 4). After that we added a small amount of Phenol Red dye powder to the starting point (indicated by A in the Figure 4). The shortest path was visualized within 60 s similarly to our previous setups due to induced Marangoni flow.

## CONCLUSIONS

In summary, we have shown a simple method for maze solving and visualizing all possible paths using fatty acid chemistry. Contrary to a previous study,<sup>11</sup> in which a chemotactic fatty acid droplet can solve the maze finding the shortest path due to releasing a surface active compound and creating its own convective flow in a pH gradient that is responsible for the motion of the droplet,<sup>25,26</sup> here we described a system where the dye particles can move passively towards lower pH at the liquid-air interface due to

a generated Marangoni flow. Meanwhile, these particles dissolve into a liquid phase and show the visualized solutions in a maze. Our system also differs from the maze solving method described in Ref [10], in which a submerged polymer hydrogel in a bulk liquid phase could find the shortest path in a microfluidic maze driven by the liquid-mixing-induced Marangoni effect. Our method is based on “non-local approach” as opposed to the local algorithms which must use either some stochastic kicks to get out of the local minima or the “wall following” algorithms, which are extremely slow. The global approach requires the knowledge of the topology of the maze, which is often unrealistic. Here this problem can be overcome by exploiting information from a globally imposed field. This unconventional computational approach provides a flexible way to explore not only the shortest path, but also all existing paths in parallel through a maze. A further advantage of this method is that the process is fast compared to other chemical-based computing methods and can also visualize the paths, so additional post-processing techniques are not required.

## **ASSOCIATED CONTENT**

### **Supporting Information**

Movie shows the generated Marangoni flow induced by an acidic gel block in a narrow channel (thickness of 2 mm) containing alkaline solution of a fatty acid. This material is available free of charge via the Internet at <http://pubs.acs.org>.

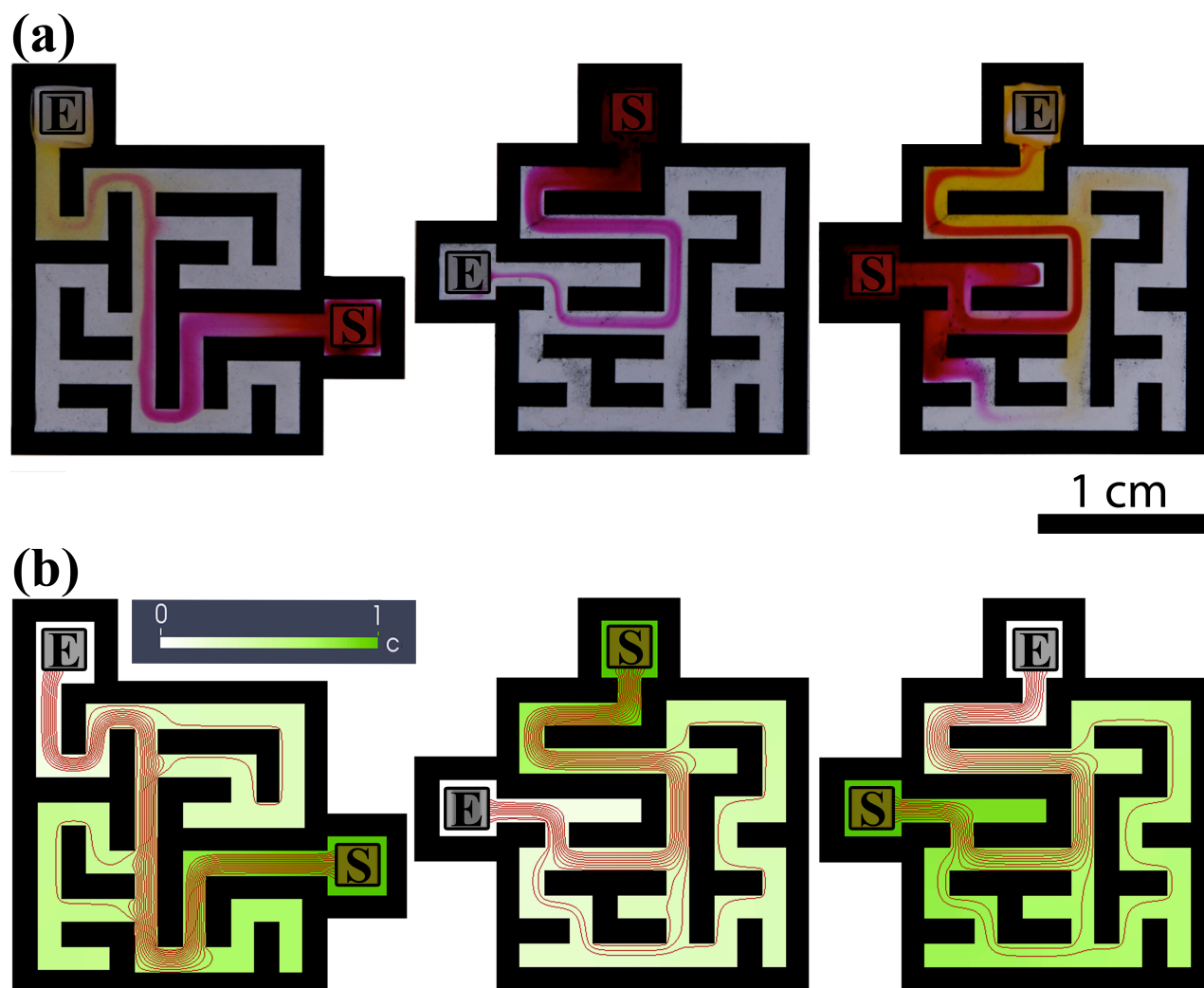
## **ACKNOWLEDGMENT**

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The authors declare no competing financial interest.

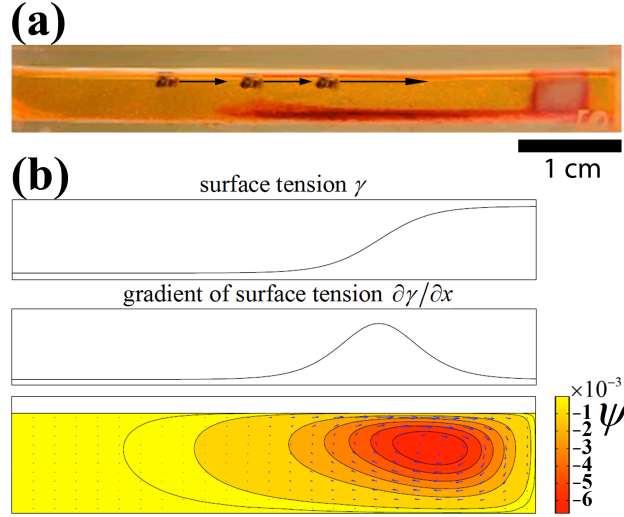
## **FIGURE CAPTIONS**



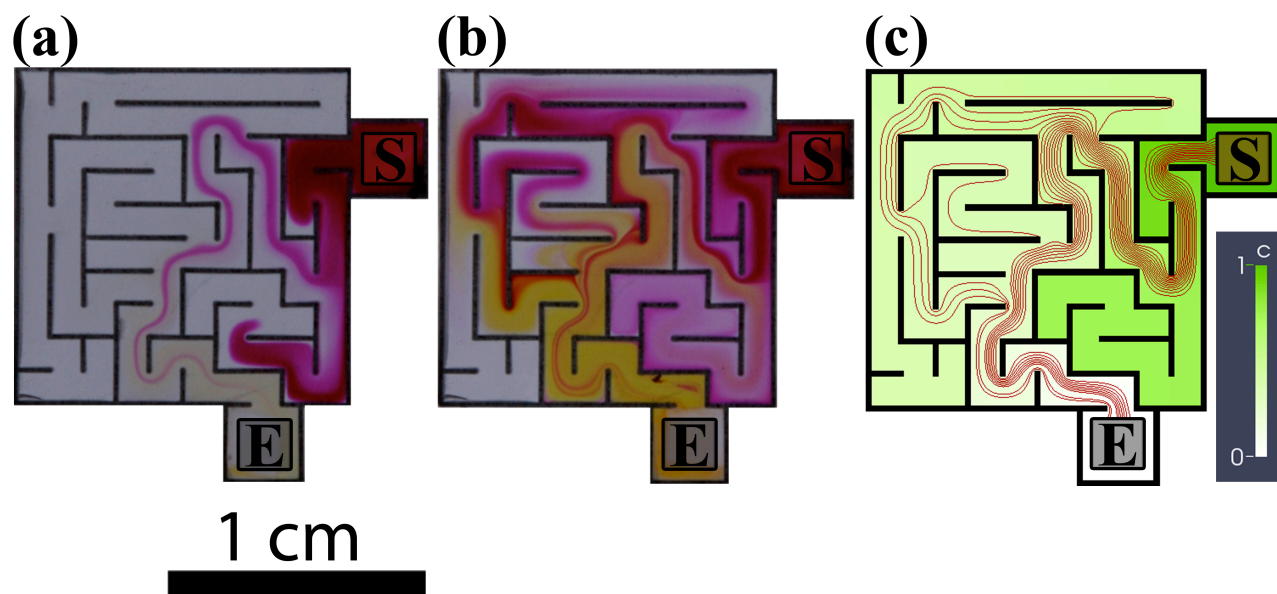
**Figure 1 (a)** Maze solving and finding the shortest paths in mazes filled with an alkaline solution of 2-Hexyldecanoic acid. Position of the gel soaked with acid at the exit of the mazes is indicated by letter E. Letter S shows the entrances of the maze, where Phenol Red dye particles are added. Marangoni flow induced by pH gradient carries particles towards the acidic region at the liquid-air interface, and the shortest path is explored and visualized due to water solubility of the dye. The yellow color near the exit originates from the color change of Phenol Red dye that has red color in alkaline ( $\text{pH} > 8.2$ ) and yellow color in acidic ( $\text{pH} < 6.8$ ) solutions. **(b)** Visualization of the shortest paths by streamlines that follow the highest concentration gradient in mazes. Stationary concentration distribution was calculated using the diffusion equation with  $c = 1$  (at the entrance) and  $c = 0$  (at the exit). This corresponds to the



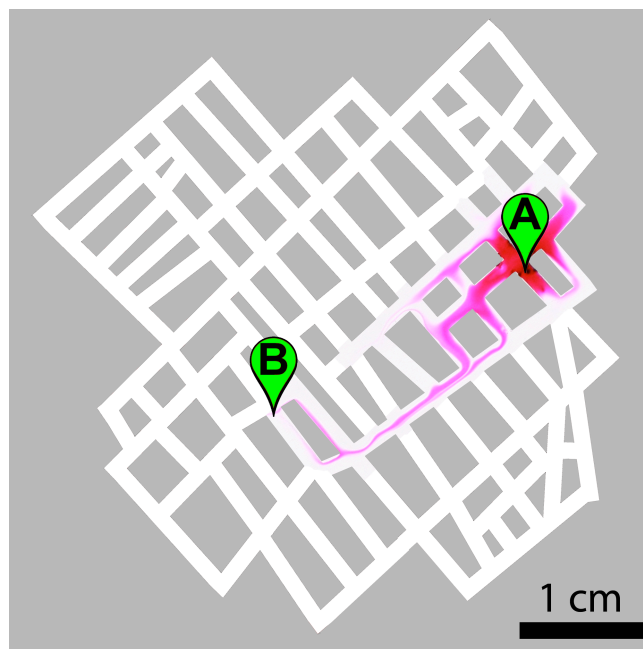
concentration of the deprotonated form of the 2-Haxydecanoic acid that is high at the entrance (basic medium) and low at the exit (acidic medium).



**Figure 2 (a)** Experimentally observed Marangoni flow induced by an acidic gel block in a narrow channel (thickness of 2 mm) containing an alkaline solution of 2-Hexyldecanoic acid. Acid diffused from the gel creates a pH gradient at the liquid-air interface that changes the concentration of the deprotonated 2-Hexyldecanoic acid. This results in a surface tension difference and generates a fluid flow towards lower pH region at the interface. This flow carries a small piece of PDMS ( $1.0 \times 0.5 \times 0.5$  mm) at the liquid-air interface. We added pH-Neutral Red dye for visualization of the fluid flow in the medium. Video1 accompanying the figure is included in the SI. **(b)** Simulated Marangoni flow by solving numerically the Navier-Stokes equation using a preexisting surface tension at the liquid-air interface (Eq. 5 in the text) with  $Re = 800$ ,  $a = 10$  and  $x_0 = 0.2$ . The values of the dimensionless surface tension are between 0.1 and 1.1.



**Figure 3** (a) Maze solving and finding the shortest path by Phenol Red dye particles after  $\sim 10$  s in a maze filled with an alkaline solution of 2-Hexyldecanoic acid. (b) All possible paths can be also explored and can be fully visualized after  $\sim 60$  s. Position of the gel soaked with acid at the exit of the mazes is indicated by letter E. Letter S shows the entrances of the maze, where dye particles are added. (c) Visualization of the shortest path and all possible paths by streamlines that follow the highest concentration gradient in mazes.



**Figure 4** Finding the shortest path between two points in a channel network (made from PDMS) based on the street map of downtown of Budapest filled with an alkaline solution of 2-Hexyldecanoic acid. Position of the gel soaked with acid (end point) is indicated by letter B. Letter A shows the starting point, where Phenol Red dye particles are added.

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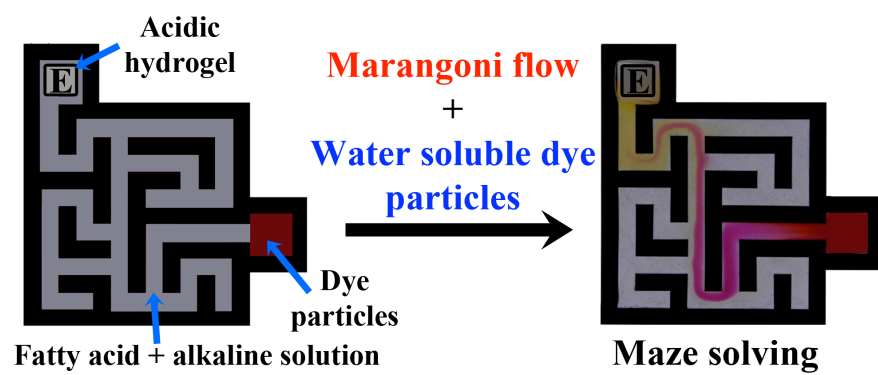
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